characterized by the complete set of Bekker soil values, it is found that several variables must be controlled independently. However, if the soil is a nonviscous, cohesionless soil, such as sand or gravel whose bulk modulus is neglected, it is only necessary to change the vehicle speed to determine the effect of $g$. Since many lunar soil investigators have postulated a cohesionless type of soil, the technique of controlling vehicle speed may be all that is necessary. However, consideration is being given to production of artificial soils whose mechanical properties can be varied independently.

In addition, test data are presented for a vehicle model operating in $\frac{1}{4}$-in. gravel, which is a nonviscous, cohesionless soil. Test results indicate that the ratio of drawbar pull to vehicle weight is essentially independent of $g$ for a given % slip. Therefore, it is concluded that, if the vehicle model tested were operating on the lunar surface in a soil with mechanical properties similar to those of the soil used, the vehicle's ability to tow, bulldoze, or accelerate would be reduced by a factor of six, whereas its ability to climb a hill would be unaffected.


**References**

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# Schlieren and Shadowgraph Studies of Hybrid Boundary-Layer Combustion


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A SERIES of experiments has been performed to check the validity of the hybrid model and analysis of Marxman and Gilbert.[1] The structure of the boundary layer was examined, and the location of the flame zone within the boundary layer was determined.

A small scale wind tunnel was used in the test, and the experiment is shown schematically in Fig. 1. Plexiglas (Polymethyl Methacrylate) slabs 1-in. wide and $\frac{1}{4}$-in. high $\times$ 6-in. long were mounted in the test section and ignited in an oxygen stream. Shadowgraph and schlieren studies with a horizontal knife edge then were made of the boundary layer formed over the slab, a system that approximates two-dimensional flow.

A natural parameter that enters into the theoretical development of Marxman and Gilbert[1] is the location of the flame zone within the momentum boundary layer. This parameter was investigated and a 3-msec exposure schlieren photograph appears in Fig. 2. The edge of the thermal boundary layer can be distinguished easily, and if the Lewis and Prandtl numbers are assumed to be near unity, then the edges of the concentration boundary layer, the thermal boundary layer, and the momentum boundary layer are located at the same point.

In analyzing Fig. 2, it is assumed that the gas-density gradients, which cause the schlieren system to respond, are related to temperature variations alone. The density also

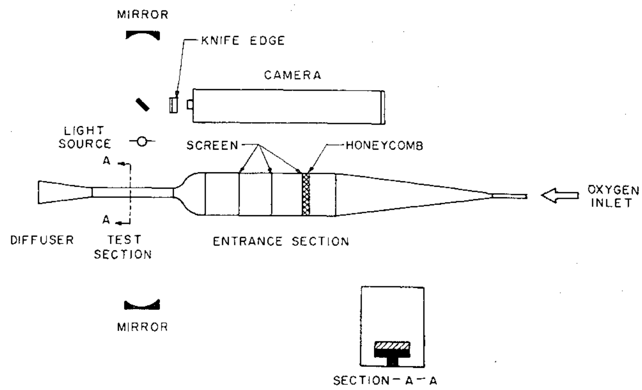
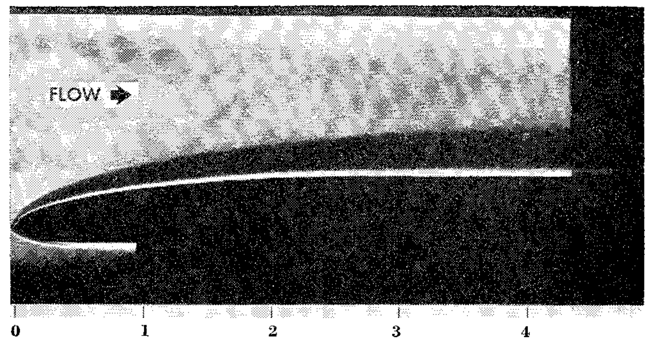



**Fig. 1 Experimental apparatus.**




$Re = 2 \times 10^4$/in.

**Fig. 2 Schlieren photograph of the combustion boundary layer (exposure time 3 msec).**

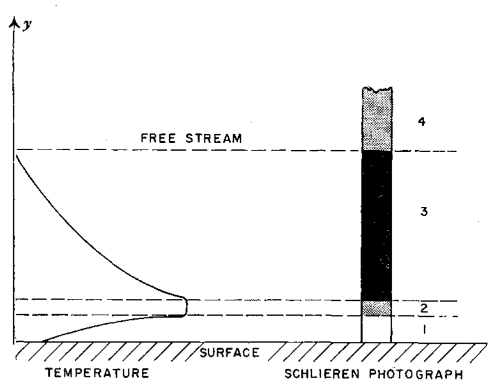



**Fig. 3 Interpretation of a typical schlieren photograph.**

depends on the local molecular weight of the mixture, but the variations in molecular weight are relatively small.

An interpretation of the schlieren photographs appears in Fig. 3. In order to analyze the photograph, it will be noted that the angular deflection of a ray of light $\epsilon$ that passes through a schlieren field is proportional to the density according to the relation[2]

$$|\epsilon| \sim (d\rho/dy) \sim (1/T^2)(dT/dy)$$

The gray region 4 in the "freestream" signifies an area of essentially no temperature gradients ($\epsilon \approx 0$). The darker region 3 indicates large negative temperature gradients whereas the lighter region 1 results from strong positive gradients. Dividing these two zones is a second gray region 2, where the temperature goes through a maximum ($\epsilon \approx 0$). The flame lies in this zone.

The uniformity in region 2 gives an indication of the finite thickness of the flame, and, in the present analysis, the flame zone is assumed to lie in the center of this region. Measurements made on this basis indicate that the height of the flame zone above the surface is about $\frac{1}{10}\delta$ (where $\delta$ is the measured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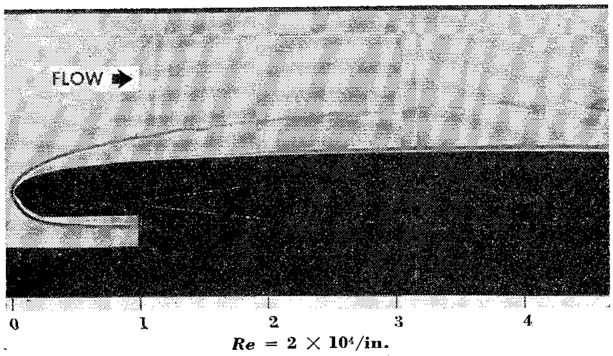


**Fig. 4 Instantaneous shadow photograph of the combustion boundary layer (exposure time 5 μsec).**

boundary-layer thickness). This distance is not influenced strongly by the Reynolds number. These results are in very good agreement with the behavior predicted by Marxman and Gilbert.[1]

A portion of the schlieren light passing through the test section was blocked off near the downstream end of the slab in Fig. 2, and a photograph of the flame alone was thus obtained. The flame is seen as a luminous streak in the dark right-hand region of Fig. 2. The position of the flame predicted by the schlieren interpretation compared accurately with this direct observation. (Compare Figs. 2 and 3.)

The instantaneous structure of the boundary layer is shown in a 5-μsec exposure shadowgraph that appears in Fig. 4. Transition from a laminar to a turbulent structure occurs at very low Reynolds numbers, apparently even lower than those expected when there is equivalent mass injection without combustion. In Fig. 4 large scale turbulence is visible at the edge of the boundary layer at a Reynolds number of about $2 \times 10^4$. Measurements in boundary layers without blowing[3] indicate that transition occurs upstream of the point where this edge turbulence is present. Therefore, transition probably takes place very near the leading edge in Fig. 4 or at a Reynolds number somewhat less than $2 \times 10^4$. The effect of combustion may be inferred by noting that Mickley and Davis[4] have reported a transition Reynolds number of $6 \times 10^4$ for a noncombustible boundary layer under similar freestream and surface blowing conditions.

The increase in thickening of the boundary layer due to blowing can be observed in Fig. 4 (near the leading edge) by comparing the thickness of the boundary layer on top of the plate, where there is blowing, with that on the bottom, where there is none.

The forementioned experiment simulates a two-dimensional hybrid motor operated at a low-oxidizer flow rate. In a practical range of oxidizer flows for hybrid combustion the transition point occurs even closer to the leading edge, indicating that it is reasonable to approach the problem of hybrid combustion solely from the point of view of a turbulent boundary layer, as stated by Marxman and Gilbert.[1] Further studies are being conducted and will be available for publication at a later date.

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# Skin Friction Exerted by a Compressible Fluid Stream on a Flat Plate

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## 1. Comparison of Existing Theories with Experimental Data

THERE exist about 30 published theories for the calculation of the drag on a flat plate under conditions yielding a turbulent compressible boundary layer. Most of these have been shown to give fair agreement with some of the experimental data. However, it hitherto has been impossible to decide which of the theories gives the best agreement with the complete body of available experimental data.

The authors accordingly have collected as many experimental data as possible (491 values of drag coefficient with associated Reynolds numbers, Mach numbers, and ratios of wall temperature to mainstream temperature) from 22 references. Computer programs have then been written for each of 20 theories; with their aid, the root-mean-square value of the proportional error in drag coefficient has been computed for each theory by reference to all 491 experimental conditions. By proportional error is meant, the experimentally measured

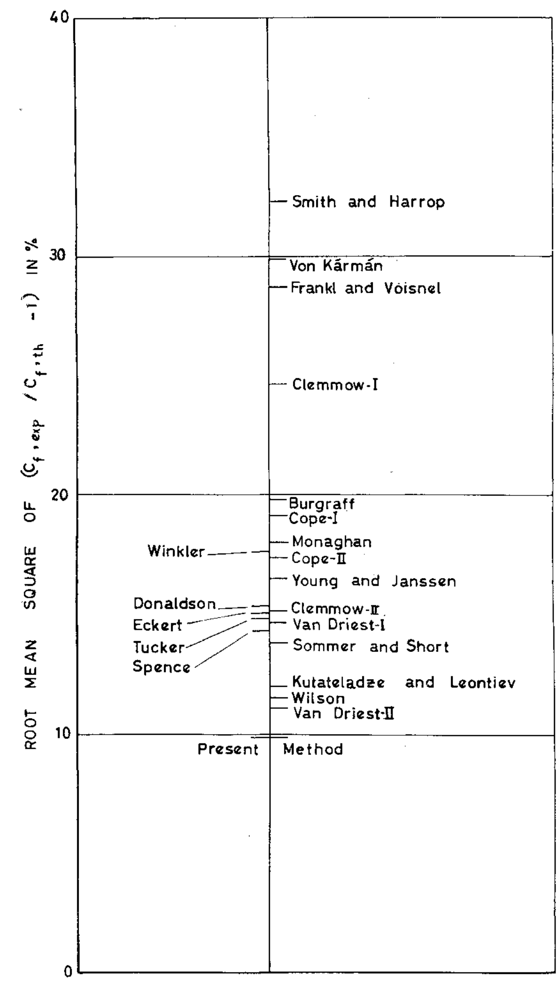


**Fig. 1 Comparison of 20 theories with experimental data from 22 sources.**

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